

**Pump Scheduling for Uncertain Electricity Prices<sup>^</sup>**  
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<sup>^</sup> Please cite as: Eck, B; McKenna, S; Akrhiev, A; Kishimoto, A; Palmes, P; Taheri, N; and van den Heever, S. (2014) "Pump Scheduling for Uncertain Electricity Prices" World Environmental and Water Resources Congress.

## **ABSTRACT**

Water utilities have optimized pump schedules to take advantage of day/night electricity pricing plans for several decades. As intermittent renewable energy sources such as solar and wind power provide an increasingly large share of the available electricity, energy providers are moving to dynamic pricing schemes where the electricity price is forecast 24 hours in advance on 30-minute time steps. The customer only knows the actual price several days after the electricity is used. Water utilities are uniquely positioned to take advantage of these dynamic prices by using their existing infrastructure for pumping and storage to respond to changing costs for power. This work develops an operational technique for generating pump schedules and quantifying the uncertainty in the cost of these schedules. With information about the pumping schedules and the distribution of possible costs, a system operator can pump according to her desired level of risk.

To develop this information, a representative sample of electricity price forecasts covering nearly the full range of possible price curves must be created. Forecasts from the energy supplier and historical data on actual prices are used to condition stochastic sampling of daily energy price trajectories using covariance decomposition methods. From this ensemble of realizations, electricity price profiles are classified into a handful of scenario classes. The optimal pumping schedule for each price class is then computed. Once the pumping schedule is known, the price of that schedule is evaluated against all other price classes to determine the robustness of the schedule.

The method is applied on a simple real-world network in Ireland. In this application, electricity prices vary every half hour and range from 5 to 262 €/mWh. Optimizing the pumping schedule proved to be the slowest step in the process so selection of proper price scenarios on which to generate the schedule was critical to obtaining results in an operational time-frame.

## **INTRODUCTION**

Water utilities have optimized pump schedules to take advantage of day/night electricity pricing plans for several decades. As intermittent renewable energy sources such as solar and wind power provide an increasingly large share of the available electricity, energy providers are moving to dynamic pricing schemes where the electricity price is forecast 24 hours in advance on 30 minute time steps. The customer only knows the actual price several days after the electricity is used. Water

utilities are uniquely positioned to take advantage of these dynamic prices by using their existing infrastructure for pumping and storage to respond to changing costs for power.

Techniques to minimize pumping costs in water systems have received considerable study in the literature. Since the early work in the mid 1970s both mathematical optimization and meta-heuristic approaches to the problem have been proposed. Math programming methods include dynamic programming (Sterling and Coulbeck 1975b), linear programming (Jowitt and Germanopoulos 1992), mixed-integer linear programming (Little and McCrodden, 1989). Meta-heuristic approaches to optimization have also been applied including hierarchical decomposition (Sterling and Coulbeck 1975a), genetic algorithms (Van Zyl et al 2004), ant colony optimization (Lopez-Ibanez et al 2008) and simulated annealing (McCormick and Powell 2004). The most recent approaches, termed hybrid methods, combine math programming techniques with heuristics to arrive at good solutions quickly. Giacomello et al (2013) solve a linear programming relaxation and then use this solution to start a greedy algorithm.

Much of the existing literature considers that the energy price is given as an input. So far, the principle way of including uncertainty in electricity prices is through a maximum demand charge. Under a maximum demand tariff, the total energy cost depends on both on the time of consumption and the maximum power demand over a billing period such as one month. The max power demand depends in turn on the water demand. An optimization method for this problem is proposed by McCormick and Powell (2003). Their method finds a maximum power demand and uses this to constrain the daily operational schedule.

The present paper focuses on the daily pumping schedule, but with the consideration that the energy price fluctuates at 30 minute intervals. These fluctuations in energy price occur when renewable but variable power sources such as wind energy connect to the grid (Garcia-Gonzalez et al 2008). Historical data on actual prices are used to condition stochastic sampling of daily energy price trajectories using covariance decomposition methods. From this ensemble of realizations, electricity price profiles are classified into a handful of scenario classes. The optimal pumping schedule for each price class is then computed. Once the pumping schedule is known, the probability distribution of cost for that schedule is evaluated using Monte Carlo methods. With information about the pumping schedules and the distribution of possible costs, a system operator can pump according to her desired level of risk.

## **METHODS**

To develop an optimal pump schedule over uncertain electricity prices, methods are applied from signal processing, machine learning, math optimization and statistics. The overall technique proposed here proceeds through several steps:

1. Collect a sample of real price trajectories that represent the time of interest.
2. The sample size may be small and thus may not fully describe the possible price scenarios, so generate 1000 (in this case) random samples of price profiles that are statistically similar to the observations

3. Divide these 1000 random samples into 10 clusters and identify the medoid trajectory of each
4. Compute the optimal pumping schedule for each cluster using the medoid price
5. Estimate the probability density function of daily pumping cost for each schedule using the set of random samples
6. Compute the desired objective value from this bootstrapped probability density function

Further details on generating correlated random samples, finding optimal pumping schedules and grouping price trajectories into clusters are given in the following subsections.

**Electricity Price Sampling.** In order to generate a large number of price scenarios, it is necessary to expand the size of the observed price scenario data set. This expansion is accomplished by random sampling from the observed price distribution at each 30-minute time step while maintaining the temporal correlation of the observed price scenarios across time steps. Each day of observed prices,  $x$ , is stored as a column in matrix  $X$ . The sampling method proceeds through a series of steps:

1. The temporal correlations in the observed price data are removed using whitening. Each sample,  $x$ , is centered and decorrelated using:

$$y = W(x - \mu) \quad (1)$$

where  $\mu$  is the vector of row means of  $X$  and the whitening matrix  $W = \Delta^{-\frac{1}{2}}U^T$  is computed from a eigendecomposition of the sample covariance matrix  $C$ . In the case that  $C$  is symmetric positive definite,  $\Delta$  is the diagonal matrix of eigenvalues and  $U$  is the corresponding matrix of eigenvectors.

2. The covariance matrix defined by the observations will not necessarily be positive-definite, and, due to noise in the data, smaller eigenvalues can be near zero. A reduction in the dimensionality by removing the lowest eigenvalues (summing to less than 5% of the total spectral energy) eliminates these issues and guarantees a positive definite covariance matrix for use in the inverse whitening transform. This procedure results in new matrices  $\tilde{\Delta}_{k \times k}$  and  $\tilde{U}_{d \times k}$  of smaller size.
3. Generate random uncorrelated samples by selecting at random a value,  $y_i$ , and replacing that value with a Gaussian deviate drawn from a distribution with mean,  $y_i$  and standard deviation chosen as  $1.06\sigma N^{\frac{1}{5}}$  where  $N$  is the number of samples and  $\sigma$  is the sample standard deviation (Epanechnikov 1969). In this case  $\sigma = 1$  due to the whitening transform. The simulated vector is then transformed back to the original observation space using the inverse whitening transform with the new matrices from Step 2.

4. Check the new sample  $\tilde{x}$  for feasibility. Discard the sample if any values are outside the range of the original training set.

Testing of this process has shown that the generated scenarios reproduce the observed distributions at each time step and also reproduce the measured temporal correlation.

**Clustering.** The clustering implementation uses PAM (Partition Around Medoids) developed by Kaufman and Rousseeuw (1990) with  $k=10$  medoids and euclidean dissimilarity metric.

**Optimal Pump Scheduling.** The optimization problem considered here is to schedule pumps to minimize electricity costs. The formulation given here considers constant speed centrifugal pumps. The technique of piecewise linearization is used to model nonlinear pump and pipe hydraulics in a mixed integer linear program (MILP).

A water distribution network comprises  $N_n$  nodes connected by  $N_l$  links. A subset  $y$  of the nodes are tanks and a subset  $p$  of the links are pumps. The decision variables for each time  $t$  in the planning horizon are the hydraulic head  $h_{i,t}$  at each node, the flow rate  $q_{ij,t}$  through each link, and the schedule  $s_{p,t}$  for each pump. The objective function is defined as the total cost of electricity for operating  $N_p$  pumps over  $N_t$  time periods each with duration  $\Delta t$ :

$$\text{Min} \sum_{t=1}^{N_t} \sum_{p=1}^{N_p} \frac{C_t \gamma}{\eta_p} \Delta h_{p,t} q_{p,t} \Delta t \quad (2)$$

The cost of electricity in €/kWh,  $C_t$ , may be different for each time period. The total head delivered by a pump is  $\Delta h_{p,t}$  and the volumetric flow rate is  $q_{p,t}$ . The pump efficiency is  $\eta_p$  and the specific weight of water is  $\gamma$ . The minimization is carried out under the hydraulic and operational constraints on the network. Energy conservation for each link between node  $i$  and node  $j$  on the network is given by a piecewise linearization of the head loss curve or pump curve:

$$h_{i,t} - h_{j,t} = \begin{cases} a_{ij}^1 q_{ij,t} + b_{ij}^1 & q^1 < q_{ij,t} < q^2 \\ \vdots & \\ a_{ij}^m q_{ij,t} + b_{ij}^m & q^m < q_{ij,t} \end{cases} \quad (3)$$

Conservation of mass around nodes with a demand  $D_{j,t}$  is given by:

$$\sum_i q_{ij,t} + \sum_k q_{jk,t} = D_{j,t} \quad (4)$$

where  $q_{ij,t}$  is the flow toward node  $j$  from node  $i$  for time period  $t$ . Mass conservation for tanks is expressed in terms of the net flow and the cross sectional area  $A$ .

$$h_{y,t+1} = h_{y,t} + \frac{1}{A \Delta t} \left( \sum_i q_{iy,t} + \sum_k q_{yk,t} \right) \quad (5)$$

Simple bounds are enforced on link flow rates and nodal heads:

$$Q_{\min} < Q_{ij} < Q_{\max} \quad (6)$$

$$0 < h_i < h_{\max} \quad (7)$$

Several operational constraints also apply to the problem. The head (and thus level) in each tank should be at least as high at the end of the planning period as the beginning:

$$h_{y,N_t} \geq h_{y,1} \quad (8)$$

The flow rate for each pump is positive while operating or zero when off:

$$0 < q_{p,t} \leq s_{p,t} Q_p^u \quad (9)$$

The number of times each pump is started during the planning horizon is limited to  $N_s$ :

$$\sum_{t=2}^{N_t} (s_{t,p} - s_{t-1,p}) \leq N_s \quad (10)$$

The optimization problem of Eqs (2) to (10) is solved using CPLEX (IBM, 2013) to find the minimum cost pumping schedules that satisfy the physical requirements of the system.

## APPLICATION

The methods outlined above were applied on a small network where electricity costs dominate operational expenses and dynamic pricing tariffs are available from the utility. This study considers pumping operations of the network for one ‘typical’ day in May 2013.

To facilitate the study a hydraulic simulation model was developed and calibrated in consultation with the utility. The system includes two pumps drawing from a single reservoir, each pumping into a storage tank approximately 50m above the reservoir. The storage tanks are connected and system demands are allocated to a single node connected to the second storage tank. The hydraulic model was applied each day for seven days; modeled tank levels were found to have good agreement with measured levels.

Dynamic electricity prices for 68-days from April to June 2013 were obtained from the electricity market. Prices fluctuate on a 30-minute basis according to supply and demand. Over the study period, energy prices ranged from 5 to 262 €/mWh and had an average value of 63 €/mWh. Prices levels were correlated to the time of day but showed considerable fluctuation.

The actual electricity price profiles were used to randomly generate a sample of 1000 similar price profiles using the methods described above. The distribution of electricity price within each half hour was compared between the randomly generated and observed electricity prices (Figure 1). The generated price profiles were similar to observed ones in terms of median value, inter-quartile range, and extreme values.

The randomly generated electricity price profiles were grouped into ten clusters and the price profile nearest the medoid of the cluster was identified (Figure 2).

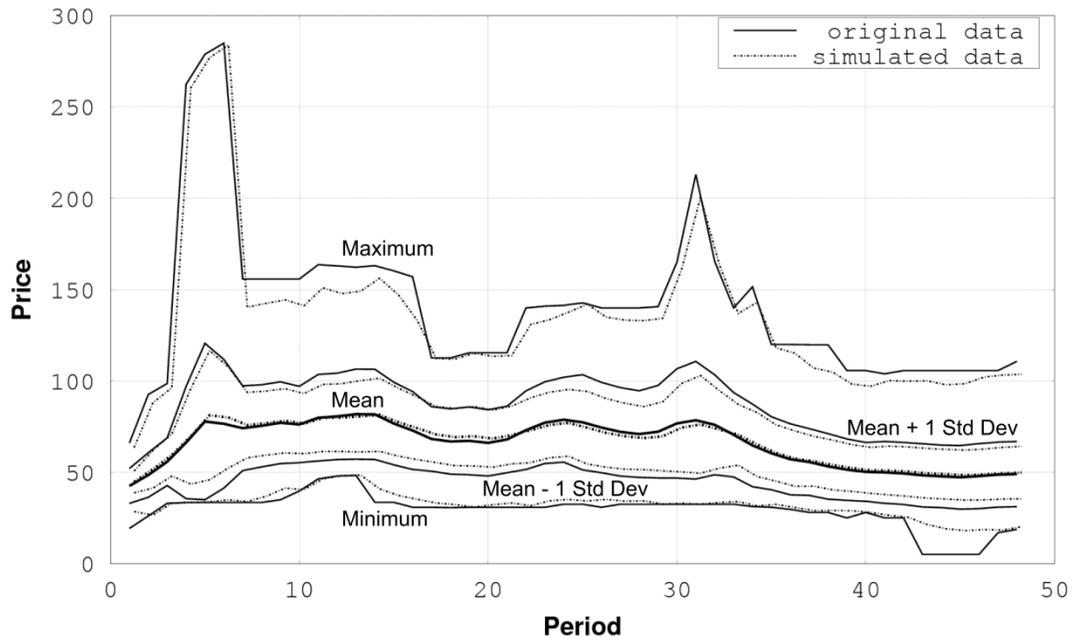


Figure 1 Comparison of randomly generated prices with actual prices. Lines are offset slightly to show differences

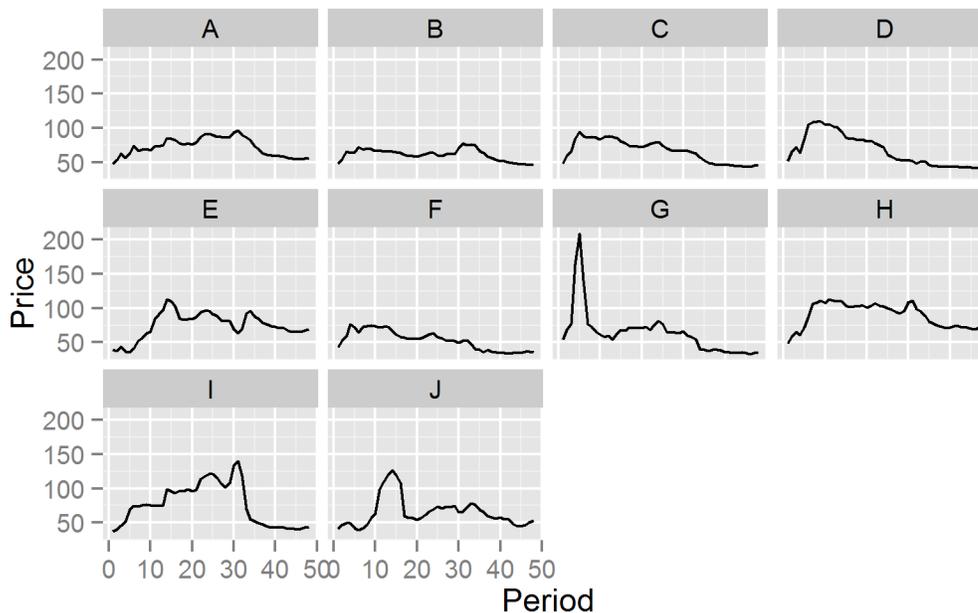


Figure 2 Medoid price trajectories for ten clusters

An optimal pumping schedule was computed for each medoid price profile. The resulting schedules (Figure 3) use a maximum of 6 pump starts per day and

emphasize pumping at night, when prices are generally lower. Between the schedules there are fluctuations in pump start and stop times according the different prices.

The uncertainty in the daily pumping cost was explored through Monte Carlo simulation. The cost of pumping schedules A-J was calculated for each of the 1000 randomly generated price profiles. Figure 4 shows the distribution of pumping cost for the optimal pumping schedule based on medoid price trajectories. In the plot, the median value is the heavy line inside a box representing the interquartile range. The whiskers show approximately 95% confidence intervals around the median and values beyond this range are plotted.

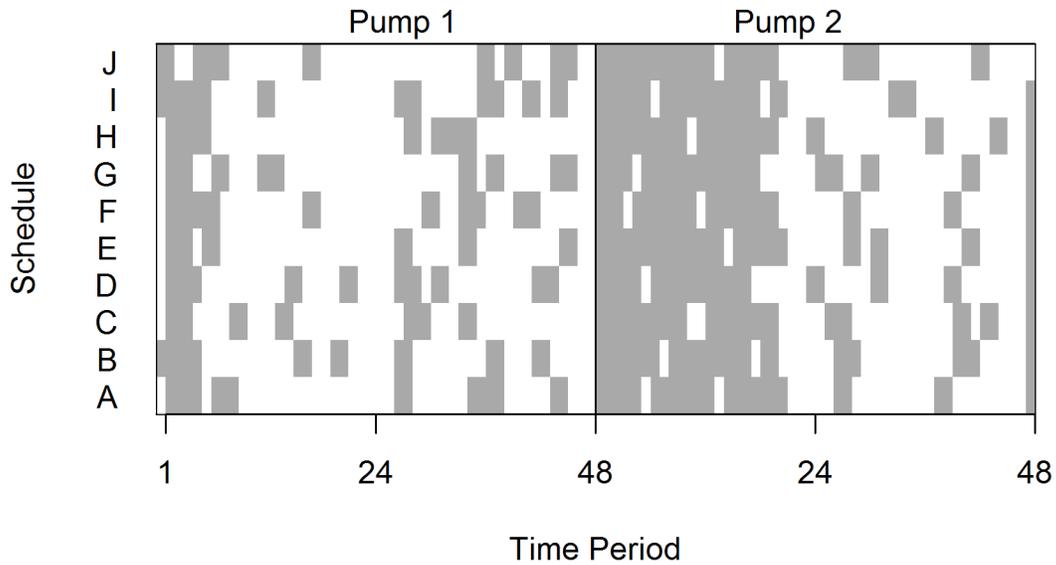


Figure 3 Optimal pump schedules for 10 medoid price trajectories

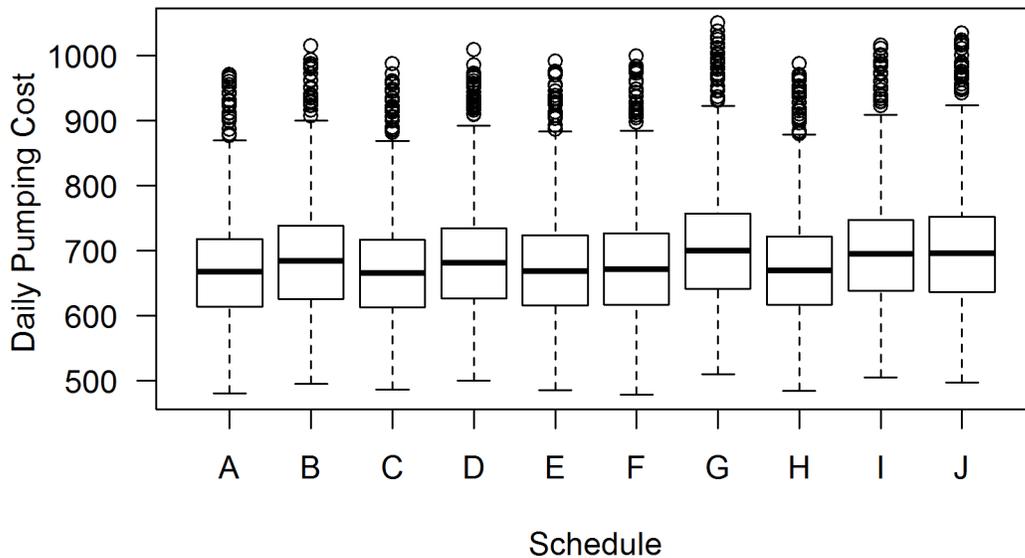


Figure 4 Boxplot of Monte Carlo results for pumping costs under different schedules. Median values shown in bold line within box for interquartile range.

Considering the foregoing analysis, which pump schedule should be selected? The appropriate pump schedule to select depends on the criterion of the system operator and the desired risk profile. Different criterion will produce different schedules (Table 1). For example, the schedule with the lowest average cost (schedule C) differs from that with the lowest maximum cost (schedule A).

**Table 1 Optimal schedules over different measures of pump schedule performance**

<b>Measures</b>	<b>Schedule</b>	<b>Value</b>	<b>Next best</b>
Lowest Average Cost	C	671	A 673
Lowest Std Error of Mean	A	2.66	C 2.69
Lowest Median Cost	C	667	A 668
Lowest inner quartile range	A	103	C 104
Lowest max cost	A	971	H 987

## CONCLUSION

This paper has combined a series of techniques to provide a method for scheduling pumps when electricity prices are unknown. By clustering energy price profiles into groups, only a modest number of pump scheduling optimizations are required. Once schedules are obtained, evaluating the cost for many possible price profiles is evaluated to estimate the cost distribution. Future work on this problem will examine the role of price predictions in selecting an optimal schedule.

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