Decomposition approach for background leakage assessment: BBLAWN instance

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ABSTRACT

This article summarizes an approach for the assessment and control of background leakage on water distribution networks. The methodology was developed for the Battle of Background leakage assessment for water networks (BBLAWN) held at the Water Distribution System Analysis Conference 2014 in Bari, Italy. The problem instance posed for the conference considers an aging water network with high levels of background leakage. A range of operational and design changes including new valves, pipes, pumps, tanks, and controls are available to reduce the expenditure needed to operate the system. Constraints are imposed on nodal pressures and tank levels to meet service level requirements. The solution methodology proposed in this paper decomposes the problem according to the type of intervention, considering each type separately. An initial diagnosis of the network informs the manner and order of evaluating the various interventions. Custom implementations of network simulation, heuristic algorithms and optimization models are used to identify improvements. The recommended program of network modifications reduced the annual cost of running the system from €4M to €1.5M and had a return on investment in network infrastructure of 430%.

Keywords: Water Networks, Optimization, Leakage

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INTRODUCTION

Many water distribution systems built in the twentieth century are experiencing challenges related to aging materials and changing demand patterns. With these changes, costs of operating the system rise and even still the system may not meet minimum service requirements. Utilities which operate such systems must decide where to invest to improve the infrastructure and how to operate the system on a daily basis. Depending on the system size and complexity as well as the deficiencies that are present, several options exist between capital improvements and operational changes. Furthermore, operational consequences of capital decisions are receiving more attention, resulting in a focus on total expenditure (Ofwat 2012).

The BBLAWN problem is to propose a methodology for recommending changes to the design and operation of a water distribution system to minimize total expenditure while meeting service requirements (Fig. 1). A sample network called C-Town is provided as an instance of an aging and deficient network. Background leakage consumes about 25% of water delivered to the network, resulting in high operational costs. The system is also unable to deliver the required minimum pressure at all nodes, even with all pumps running. Operational costs comprise energy used for pumping and an environmental penalty for leakage. Several strategies are available to reduce operational costs: installing pressure reducing valves; replacing existing pipes; installing new pipes in parallel to existing pipes; upgrading pumps; increasing storage; and permanently closing pipes. The cost of each of these options is reported in the problem statement (Giustolisi et al. 2014). A summary of approaches to the problem reported in this special issue is given by Giustolisi et al. (2015).

Control of background leakage is a difficult problem in general because leakage locations may be unknown and because providing a minimum pressure means that background leakage will not be eliminated completely. Several details of the C-Town instance are even more challenging. The model of Germanopoulos and Jowitt (1989) was chosen to represent background leakage at *pipe* level.

$$Q_{leak} = \beta L P_{avg}^{\alpha} \tag{1}$$

The volumetric flow rate of leakage on a pipe Q_{leak} depends on the average pressure over the pipe P_{avg}, the pipe length L, leakage coefficient β and leakage exponent α, all in suitable units. Pipes with negative average pressure are assigned zero leakage. A simulator for networks subject to such leakage was not initially available. In addition to high leakage, the C-Town network is characterized by large differences in elevation. In three district metered areas (DMAs) pumping is needed to meet minimum pressure requirements due to tank elevations. Several alternative strategies were available to improve the network and these were combinatorial in nature and related by the non-linear hydraulic simulation. In order to arrive at subproblems of a tractable size, a decomposition method for the problem of background leakage control is proposed.

Decomposing a large problem into smaller pieces is a well established technique in the optimization of water systems. Decompositions along spatial, temporal or

Figure 1. Problem summary (Reprinted from Eck et al. 2014, with permission)

minimize	annual costs = energy + leakage + capital improvement
s.t.	minimum pressure of 20m at demand nodes, positive pressure for other nodes;
	storage tanks maintain a level above zero and recover to their original level;
where	energy cost varies over time and pumping efficiency follows a parabolic curve;
	leakage is proportional to pressure to a power and leakage is valued at €2 / m3;
	annual costs are provided for various capital improvements:
	replacing and paralleling pipes, adding pumps,
	adding hydraulic control valves, enlarging tanks;
	pumps are controlled by tank level.

other dimensions are available for several problems. On the optimal design of water networks, decomposition approaches have been proposed by Fujiwara and Khang (1990), Kessler and Shamir (1991), Eiger et al. (1994), and Loganathan et al. (1995). For large non-linear water management models, a piece-by-piece approach is suggested by Cai et al. (2001). Distribution systems with multiple water sources may be optimized using the decomposition and multi-stage approach of Zheng et al. (2013). The problem of pump scheduling is decomposed by Ghaddar et al. (2015). The decomposition proposed below is a procedure for dissecting the problem of background leakage assessment and control. To the authors knowledge it is a novel approach to that problem. The approach is also summarized in a conference paper (Eck et al. 2014). The treatment here expands the presentation of several elements and provides more detail on the trajectory of the objective value through the solution process.

METHODS

The overall method for background leakage assessment and control is a decomposition into smaller more tractable subproblems. Such a decomposition ignores some interaction between decisions, likely resulting in a higher objective value than considering the whole problem at once. Decomposition was selected to facilitate development in parallel. Taking an incremental approach also yields insight on the contribution of each intervention.

The proposed solution approach proceeds through several steps:

0. Perform a preliminary diagnosis of the system based on hydraulic simulation.
1. Select locations for new pressure reducing valves using mixed integer nonlinear programming
2. Choose pipes to replace or re-size by a profit-for-cost heuristic
3. Find pump control levels using coordinate search
4. Manually check pump replacements and tank additions
5. Re-run the level control optimization to ensure feasibility

These steps cover all of the options available for the problem except installing parallel pipes and closing individual pipes. Parallel pipes were not considered because they were 20% more expensive to install and have the effect of making the system longer, which further increases leakage. Closing a few individual pipes was evaluated manually by trial and error but was not considered systematically because of the similarity with placing PRVs. Also, closing many individual pipes in a real system was judged unlikely and so effort was not devoted to developing a software component to support the intervention.

Simulating leakage at pipe level

In order to perform the network optimization, it was essential to implement the background leakage simulation model requested in the problem statement. Our approach consisted of developing a Picard iteration technique using the emitters capability of Epanet (Rossman 2000). The method iteratively finds an emitter coefficient for each node at every time step so that the emitter demand equals the node-allocated leakage based on the background leakage model. Eqs. (2) and (3) represent the leakage simulation technique in compact form.

$$\mathbf{d}_n^{(k+1)} = \frac{1}{2}\mathbf{A}_{np}\left[\boldsymbol{\beta}_p \odot \mathbf{l}_p \odot \left(\frac{1}{2}\mathbf{A}_{np}^{\top}\mathbf{p}_n^{(k)}\right)^{\odot\alpha}\right] \tag{2}$$

$$\mathbf{c}_n^{(k+1)} = \left[\ \mathrm{diag}\left(\mathbf{p}_n^{(k)\odot\frac{1}{2}}\right)\right]^{-1}\mathbf{d}_n^{(k+1)} \tag{3}$$

where k is the iteration level, $\mathbf{d}_n$ is the vector of leakage allocated to the nodes, $\boldsymbol{\beta}_p$ is the vector of pipe leakage coefficients, $\mathbf{l}_p$ is the vector of pipe lengths, $\mathbf{A}_{np}$ is the un-oriented incidence matrix, $\mathbf{p}_n$ is the vector of nodal pressures, α is a leakage exponent, $\mathbf{c}_n$ is the vector of emitter coefficients, and $\odot$ represents the Schur-Hadamard or element-wise product. This approach is similar to that of Jun and Guoping (2013), except that pressure dependent demands were not specified in this problem and so were not implemented.

The following process was implemented using the Epanet toolkit, avoiding the need to modify the source code.

0. Simulate the network
1. return nodal pressures $\mathbf{p}_n^{(k)}$ and emitter coefficients $\mathbf{c}_n^{(k)}$
2. Calculate the node leakage $\mathbf{d}_n^{(k+1)}$ (Eq. 2)
3. Calculate the emitter coefficients $\mathbf{c}_n^{(k+1)}$ (Eq. 3)
4. Calculate the RMSE of the error $(\mathbf{c}_n^{(k+1)} - \mathbf{c}_n^{(k)})$
5. *If* RMSE $\leq$ tolerance then move to the next time step, *else* $k = k + 1$ and *go to* 0

Optimal Placement of PRVs

Locations for new pressure reducing valves were found using mixed-integer non-linear programming (MINLP). Comparing to heuristic methods, the chief advantage of MINLP is obtaining solutions with local optimality. Locations for new pressure reducing valves were found by solving the following optimization model (Eck et al. 2014).

$$
\begin{aligned}
\text{(VP-MINLP) min} \quad & \sum_k Q_k^{\text{leaks}} && \text{(4a)} \\
\text{such that} \quad & \sum_m Q_{m,i} - \sum_l Q_{i,l} = d_i + 0.5 \sum Q_{i,j}^{\text{leaks}} && \text{(4b)} \\
& Q_{i,j}(p_i + e_i - p_j - e_j - h_f(Q)_{i,j}) \geq 0, \quad (i,j) \in L \setminus P && \text{(4c)} \\
& p_i + e_i - p_j - e_j - h_f(Q)_{i,j} - M v_{i,j} \leq 0, \quad (i,j) \in L \setminus P && \text{(4d)} \\
& p_i + e_i - p_j - e_j + h_p(Q)_{i,j} = 0, \quad (i,j) \in P && \text{(4e)} \\
& 0 \leq Q_{i,j} \leq Q_{\max} \quad \text{and} \quad p_{i,\min} \leq p_i \leq p_{\max} && \text{(4f)} \\
& v_{i,j} + v_{j,i} \leq 1 \quad \text{and} \sum_{(i,j) \in E} v_{i,j} \leq N_v \quad v_{i,j} \in \{0,1\} && \text{(4g)}
\end{aligned}
$$

The problem formulation VP-MINLP models a water network as a directed graph comprised of links and nodes. Physical quantities specified at node i or j include pressure p, elevation e, and demand d. Physical quantities specified for the link between node i and node j include flow rate $Q_{i,j}$, head loss $h_f(Q)_{i,j}$, leakage $Q_{i,j}^{leaks}$ and a binary indicator for the presence of PRVs $v_{i,j}$, pumps, and check valves. The parameter M is a postive constant chosen to model the effect of placing a valve on a pipe as further discussed below. The set of links is L, the set of links with pumps is $P \subset L$, and the set of undirected edges is E. Following Jowitt and Xu (1990) leakage at pipe level is minimized in the objective function 4a and included in the mass balance constraint 4b. Energy conservation taking into account directionality of the pipe flow is modeled using a pair of constraints 4c and 4d suggested by Sherali and Smith (1997). Pipe friction is modeled using the quadratic approximation of Eck and Mevissen (2015).

When a PRV is placed on a pipe, the energy conservation equation for the direction of the valve is disabled so that the pressure at the downstream node is a free variable. The optimizer is then able to set this value at the lowest pressure that satisfies the other constraints. The resulting pressure is thus used as a valve setting. It is noted that the approach for modeling PRVs described here has limitations with regard to PRVs sharing the same downstream node. The problem was modeled in AMPL (Fourer et al. 2003) and solved using Branch and Bound and interior point techniques implemented in Bonmin (2011) and Ipopt (2011).

Pipe Replacement

As a step to reduce background leakage and to minimize the cost of operation of the C-Town network, pipe replacement and resizing is explored. According to the problem statement new pipes have better friction and leakage properties than the existing pipes. Critically, new pipes still do experience leakage in this case. The decision to replace, and possibly re-size, a pipe is performed through a heuristic inspired from the profit-to-cost approach used to find good solutions for the 0-1 knapsack problem (Dantzig 1957). In the knapsack problem addressed by Dantzig, the cost and benefit of including an item in the knapsack are independent of other items. This independence does not hold for the pipe replacement problem. Replacing a pipe might affect the system by increasing the pressure and therefore leakage on another pipe, by causing a tank to fill faster and so change the timing of pumping, or other effects. With this complexity, the approach pursued here treats one pipe at a time and updates the costs and benefits of subsequent decisions by simulation.

The ratio of leakage cost, Cl_i, to cost of replacement in kind (with the same diameter), $Crik_i$ is computed for each pipe in the network and sorted in descending order.

$$PCR_i = \frac{Cl_i}{Crik_i} \tag{5}$$

Working from the top of the list the most profitable pipe to replace is considered individually. Costs are computed for replacing the worst offending pipe with the nearest available diameter. If replacement in kind reduces the total cost, changes in pipe diameter are evaluated. All available diameters are checked by simulation and the choice with the lowest cost that also maintains feasibility with regard to minimum pressures and tank levels is selected. With the replacement made, the vector (5) is computed and sorted again. Replacement and resizing of the new worst offending pipe is considered. The process repeats until profitable replacements which maintain feasibility are not found.

Pump controls

Control of pumps is an important step in providing a solution that meets pressure and tank constraints while also lowering energy costs. Finding good values for pump level switches is not straightforward because of the interactions between system demand, electricity pricing, pump efficiency, and constraints on tank level (Fig. 2). With all of these competing effects control levels were not open to optimization directly. The solution space of control levels was discretized at 0.1 meter and explored through a heuristic similar to the coordinate search method outlined by Conn et al. (2009).

This coordinate search explores the neighborhood around the current solution by separately increasing and decreasing each control level in the current solution by the mesh size 0.1 meter. The candidate solutions are evaluated by simulation and the best feasible solution becomes the next iterate. Unlike, the coordinate search described by Conn et al. (2009) the step size is not reduced over the search process.

In the case of C-Town, eleven pumps are controlled by level switches and so the vector of states s_j is a 22 element vector. Element $j = 2i$ is the level above which pump i turns off and $j = 2i - 1$ is level below which the pump turns on. Next 44 vectors of candidate control levels are generated by adding and subtracting $\delta = 0.1m$ from the starting point. The feasibility of each candidate vector was

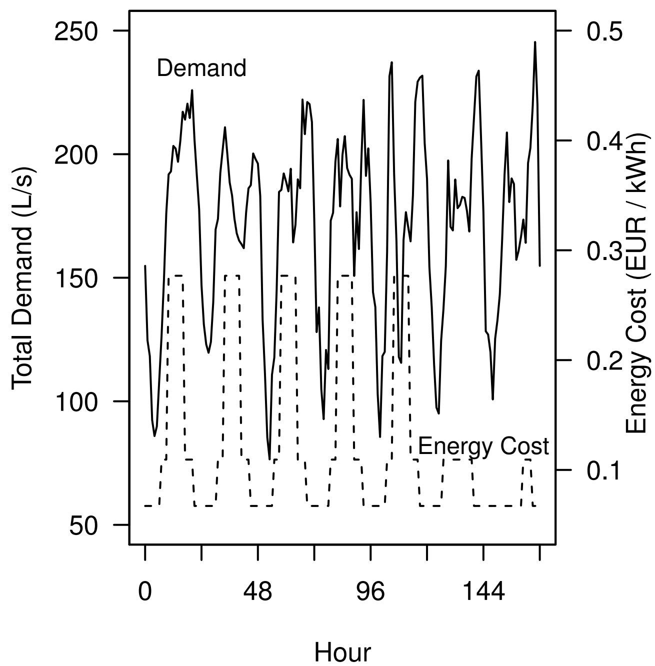

Figure 2. Pattern of system demand and energy cost over 168 hour simulation period in C-Town.

assessed by simulation. The best feasible vector was chosen for the next iteration.

Testing showed that enforcing feasibility early in the algorithm constrained its performance. Thus feasibility was enforced in stages, where given an infeasible starting point that is likely to violate all of the operational constraints, the coordinate search is allowed to move to new infeasible points until the "tanks never be emptied" constraint is satisfied, thus enforcing this constraint in all the steps that follow. Similarly, the "final level in each tank should not be less than its starting level" is enforced following a step that reaches a point that is feasible to that constraint. Lastly, the minimum pressure constraint was enforced. The algorithm was terminated by setting a timelimit of 1 hour of computational time. This timelimit was chosen after conducting initial experiments and observing that no improvements were being made to the results after 1 hour.

RESULTS AND DISCUSSION

The methods outlined above were applied to the C-Town problem instance. The resulting solution invests heavily in network infrastructure to reduce leakage. Suggested infrastructure changes include 22 new pressure reducing valves, replacement of 345 pipes and two pumps. Locations of proposed improvements are shown relative to the existing system in Fig. 3. Evolution of costs through each step are shown in Tab. 1.

Step 0: Preliminary diagnosis The C-Town network was provided without a starting point for level switches to control the pumps. In order to make an initial assessment, levels were assigned based on engineering judgement (Tab. 2) and the system was simulated with pressure driven leakage over the 168 hour planning period. The accuracy of the Epanet based fixed-point hydraulic simulator was assessed by manually checking nodes and pipes to confirm equations were satisfied and by comparing with the solution obtained from a separate simulator.

Results of the initial simulation revealed inadequate pressure control as the principal difficulty on the system. Annual leakage costs of €3.69M far exceeded the energy costs of €0.262M. Furthermore, the initial simulation did not meet the problem constraints due to insufficient pressure in some nodes (Fig. 4) and tanks which finished the simulation below their initial level. Based on this assessment, pressure reducing valves were evaluated first, followed by pipe replacements, pump and tank upgrades and level switches.

Step 1: Place PRVs The PRV placement process was carried out by DMA as described previously (Eck et al. 2014). The analysis placed valves based on conditions at 166 hours as this was the time of maximum demand (Fig. 2). A total of 22 valves were placed across the network (Fig. 3) at a cost of €0.0107M. Evaluating over the 168h period, this investment in reducing pressures lowered leakage by 22%. Energy costs also decreased by 9%. The annual cost reduced by 21% to €3.12M. Placing these PRVs generated a return on investment of 7700%. The very high return confirmed the decision to evaluate PRV placement as an initial step.

Step 2: Pipe replacement. Pipes were selected for replacement using the profit for cost approach. Where a pipe was economical to replace with the same diameter, larger and smaller diameters were also evaluated. In all, the algorithm selected 345 pipes for rehabilitation at a cost of €0.560M. A plurality of replacements, 148, kept the same size as the existing pipe whereas a smaller pipe was chosen in 101 cases. In 63 cases the diameter was increased to the smallest available size. For the remaining 33 pipes, the heuristic recommended an increase in diameter sometimes resulting in a large pipe between two smaller ones. This result arises from the order in which the heuristic replaced pipes. Early in the optimization, a larger pipe was economical and this decision was not re-evaluated unless the pipe again becomes worst in the network. Even using larger pipes in some locations, pipe replacement had a return on investment of 66% and further reduced annual costs to €2.19M.

Step 3: Level switches In the C-Town problem, electricity prices varied over time but pumps were controlled by level. From a practical perspective, the choice to control by level provides robustness against uncertain demands but also limits the optimization available from pumping when energy is cheaper. Tests showed many sets of control levels provided essentially the same electricity costs. Finding control levels which allowed tanks to recover their initial level was more difficult. A first optimization of level switches saved €0.486M.

Step 4: Pump and tank upgrades Upgrades to pumps and tanks were considered by trial and error. Replacing pumps P1 and P7 with more efficient models at a cost of €0.00847M was found to generate a return of 1300%. Increasing the size of tanks was also considered. Adding storage tank volume was found to increase leakage and electricity costs as larger tanks took more time to drain and

Table 1. Components of objective value in thousands of Euro at steps in the solution process. Costs for capital improvements are annualized over the asset's useful life.

Step	PRVs	Pipes	Pumps	Leakage	Energy	Total	Marginal Return
0	-	-	-	3,688	262	3,950	
1	11	-	-	2,873	237	3,120	7700%
2	11	560	-	1,447	207	2,193	66%
3	11	560	-	849	187	1,707	inf
4	11	560	8	730	178	1,587	1300%
5	11	560	8	683	192	1,454	inf

so maintained the system at higher pressures.

Step 5: Level switches With all of the design changes to the network in place, a final run of the level switches was carried out. The recommended set of control levels showed a trade off between energy and leakage costs. Pumping when electricity prices are lower kept tank levels higher and so increased pressure and leakage. Under the specified costs, keeping tank levels low was better. Level trajectories for all of the tanks except T3 stay above their starting point for the full simulation period (Fig. 5). These tanks remained above their starting level—rather than dropping to a lower level to reduce pressure and pumping requirements— because the algorithm did not find a set of level switches that allowed the level to drop and then recover to the initial level at the end of the 168h simulation. A different requirement, such as illustration of a steady-state level trajectory, would allow lower tank levels for part of the simulation, reducing pumping and leakage costs.

The improvements proposed here provide C-Town with a water distribution system that requires an annual cost of €1.454M and meets the required operational constraints. This annual cost includes payment of principal and interest on capital investment amortized over the life of the asset as specified in the problem statement. Despite these efforts, leakage remains the highest component of the operating cost (47%) because new pipes also leak according to the problem specification. Furthermore, the topography of the network means that some nodes continue to experience high pressure due to their elevation (Fig. 6). Even with the 22 PRVs and the other recommended improvements, the median of the minimum pressures is 39m. Adjusting settings of PRVs by time might affect some reduction, but was judged to have only marginal benefit compared to the implementation effort and was not pursued.

CONCLUSIONS

A decomposition method for background leakage assessment on water networks has been developed and applied to the C-Town problem instance posed for the Water Distribution Systems Analysis 2014 conference. The decomposition evaluates each class of improvement separately and in an order suggested by a preliminary analysis. This approach creates smaller problems which are computa-

Table 2. Level switches in meters for pump and valve control on the C-Town network

		Human judgement		Coordinate Search Algorithm	
Tank	Link	On-below	Off-above	On-below	Off-above
T1	PU1	1	6	1.5	6
T1	PU2	2	6	2.9	5.6
T1	PU3	3	6	4.3	4.7
T2	V2	2	5.5	3.3	4
T3	PU4	1	6	1.3	6.7
T3	PU5	3	6	1.7	6.7
T4	PU6	1	4	3	3.7
T4	PU7	2	4	2.5	2.7
T5	PU8	1	4	2.5	3.5
T5	PU9	2	4	3.5	4.4
T7	PU10	1	4	3.6	3.8
T6	PU11	1	5	1.2	2.6

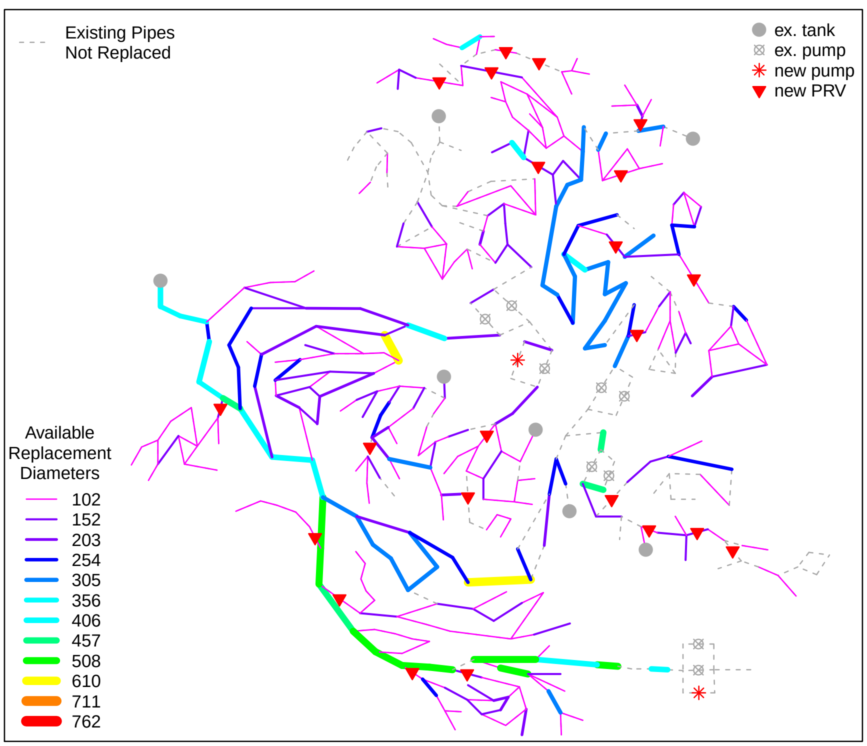

Figure 3. C-Town network for BBLAWN showing existing and proposed elements. (Adapted from Eck et al. 2014, with permision)

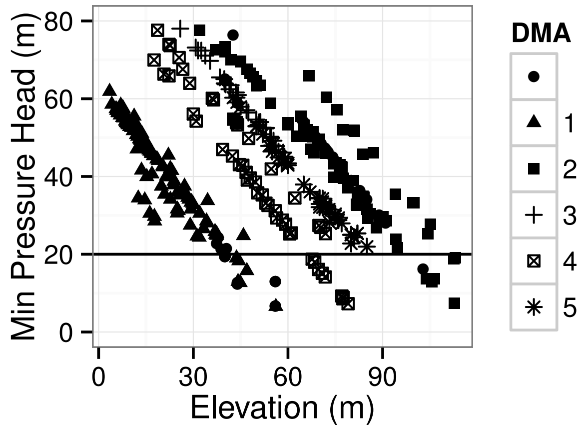

Figure 4. Nodal minimum pressure over 168h simulation period at step 0: pump controls from human judgement and no infrastructure changes.

tionally tractable, allows solutions to proceed in parallel, and adapts to problem instances where other options are available. Different techniques were applied to each sub-problem: fixed point iteration for systems of non-linear equations, mixed integer nonlinear programming for valve placement, profit-for-cost heuristics for pipe rehabilitation, random walk over a grid for pump controls. Although more sophisticated approaches exist for each of these subproblems, the chosen techniques were successful in lowering the annual cost for the system by 63%. Nonetheless, the solution could be improved by using different methods at each step.

Formulating the method as a sequence of steps also gives some visibility into the marginal contribution of each type of change to the network. For example, pipe replacements provided the largest reduction in annualized cost, but also had the lowest return on investment. Changes which required less capital such as placing pressure reducing valves and changing pump control levels provided much higher return on investment than pipe replacement, but are incapable on their own of restoring the system to sustainable operation. A wide range of options need to be considered in order to optimize capital and operating costs in real systems.

Although problem decomposition proved effective, the approach does have important limitations. Choosing the order in which to evaluate interventions is problem dependent. Evaluating the C-Town instance in a different order would give a different result. Treating interventions separately may miss profitable trade-offs between types of improvements. In addition to the order of evaluating improvements, the optimization performance may also be influenced by the initial level switches. Such a heuristic approach provides no guarantee of solution quality or optimality.

Perhaps the largest impediment to performing the analysis reported here on real systems is model and data uncertainty. The C-Town instance had thousands

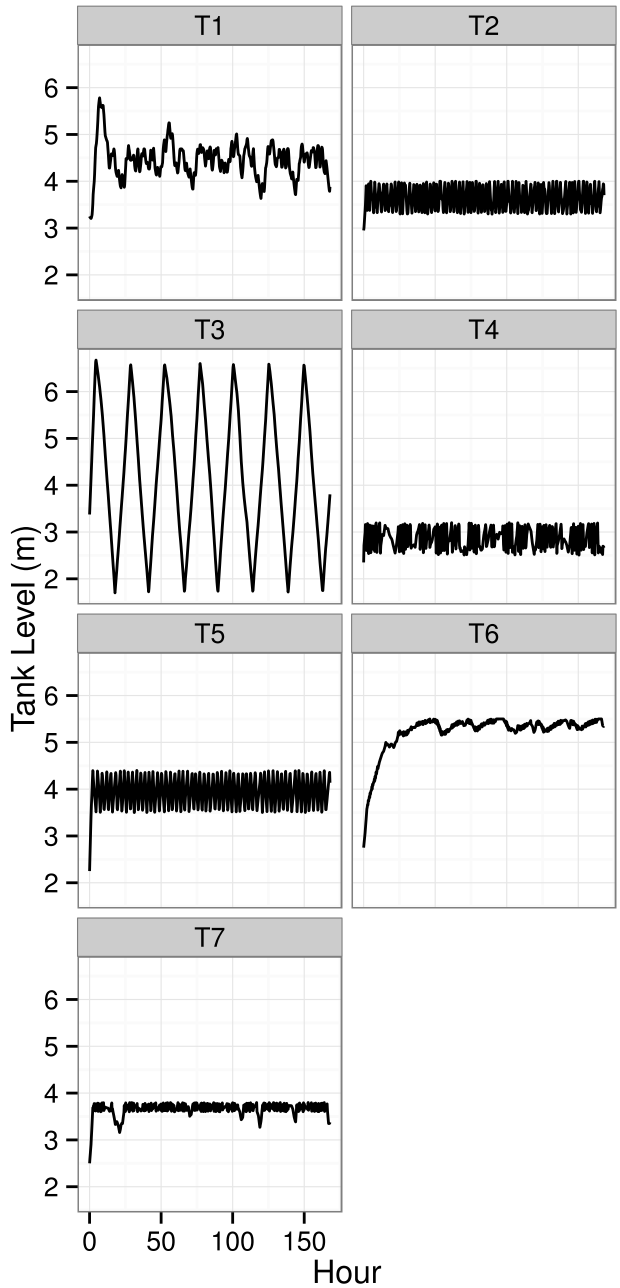

Figure 5. Storage tank levels over the 168 hour simulation period at step 5: optimized controls and infrastructure.

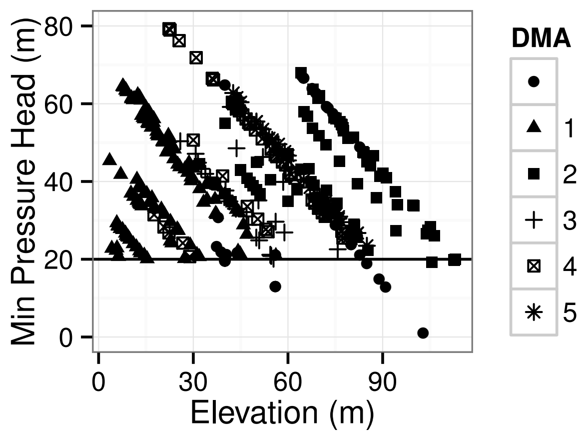

Figure 6. Nodal mininum pressure over 168h simulation period at step 5: optimizied controls and infrastructure. Nodes with demands assigned by patterns DMA1-5 have min pressures above 20m and nodes without demand have positive pressure.

of parameters which were treated deterministically. Among these, coefficients of the leakage model may be the most uncertain because parsing the difference between leakage and usage remains a challenge. Before implementing the recommendations of such a study, the robustness of the proposed solution to uncertainties in the data should be evaluated.

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